

When Skills Align: Structural Insights into Nash Stability in Non-Transferable Utility Games

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ABSTRACT

In the rapidly evolving field of Artificial Intelligence (AI), multi-agent systems have become crucial for tackling complex, distributed tasks that exceed the capabilities of individual agents. A central challenge in such systems is ensuring the stability of coalitions when agents autonomously collaborate, particularly in scenarios where utility is non-transferable. This study explores the role of skill alignment in achieving Nash stability within Non-Transferable Utility (NTU) games, where each agent's payoff depends on the coalition it forms, and no agent can transfer utility to others. We introduce the concept of a Capability Complementarity Index (CCI) to measure the degree to which agents' skills complement each other. Using this index, we examine how capability complementarity influences coalition stability in multi-agent systems. Our simulations, involving 500 agents with random capability profiles across 10 dimensions, show that coalitions with higher CCI are significantly more likely to achieve Nash stability. Additionally, the results reveal that agents tend to form larger coalitions when their capabilities align well, and such coalitions exhibit fewer defections. These findings provide valuable insights for designing decentralized AI systems, such as autonomous vehicles, cooperative robotics, and distributed sensor networks, where agents must collaborate autonomously without external authority. The study contributes to the theoretical understanding of coalition formation in multi-agent systems and offers practical design principles for creating resilient and efficient AI ecosystems. By emphasizing the importance of skill complementarity, this research helps inform the development of AI systems capable of stable, self-organizing cooperation among agents with heterogeneous capabilities.

Index Terms- Multi-Agent Systems, Nash Stability, Non-Transferable Utility, Capability Complementarity, Coalition Formation

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INTRODUCTION

In the rapidly evolving domain of Artificial Intelligence (AI), the development and deployment of multi-agent systems (MAS) have emerged as a critical area of research and application (Luzolo, Elrawashdeh, Tchappi, Galland, & Outay, 2024; Sridhar, 2022). Multi-agent systems are designed to address complex, dynamic problems that are beyond the capacity of individual agents to solve on their own (Binyamin & Ben Slama, 2022; Thummalapeta & Liu, 2023). These systems are composed of autonomous agents, each with distinct skill sets, capabilities, and local objectives (Nilsson, Danielsson, & Svensson, 2023; Maldonado, Cruz, Torres, Cruz, & Gamboa, 2024). To achieve optimal outcomes, these agents must collaborate and coordinate their actions in shared environments, often with no external authority to guide them. This opens up numerous challenges, particularly in the context of coalition formation, where agents must decide on the most efficient groupings or alliances that maximize their respective utilities.

One of the most critical issues in multi-agent cooperation, especially in decentralized AI systems, is

ensuring the stability of coalitions (Amirkhani & Barshooi, 2022; Maldonado et al., 2024). The challenge becomes particularly intricate in scenarios where agents seek to maximize their task-specific utilities without the ability to transfer or exchange payoffs between them. This kind of scenario arises in what are known as Non-Transferable Utility (NTU) games, where agents' payoffs are tied to the coalition they join, and these payoffs cannot be exchanged or redistributed among agents. In traditional game theory, transferable utility (TU) games are often used, where agents can redistribute payoffs or rewards to ensure fairness and stability in coalitions (Zhang et al., 2024; Raja & Grammatico, 2023). However, NTU games present an additional layer of complexity, as they assume that the utility is inherent to each agent's situation and cannot be altered by other agents (Ahmad & Al-Fagih, 2023; Singh et al., 2022). The focus of this study is to investigate the structural role of skill alignment in achieving Nash stability within the framework of NTU games. Nash equilibrium, a key concept in game theory, refers to a state where no agent can unilaterally improve their utility by changing their

strategy, given the strategies of others (Abedian et al., 2022). In the context of multi-agent systems, Nash stability in coalition formation is essential to ensure that once a group of agents forms a coalition, no single agent has the incentive to leave or betray the group (Sarkar, Curado Malta, & Dutta, 2022). This is particularly crucial when considering autonomous systems such as decentralized robotics, distributed sensor networks, and cooperative autonomous vehicles, where agents must operate independently while still working together to achieve a shared goal.

The concept of skill alignment plays a pivotal role in ensuring Nash stability within these multi-agent systems (Li, Zhang, Wang, Zhang, Du, Wen, & Pan, 2024; Ye, Han, Ding, & Xu, 2023). By embedding agents' skill profiles directly into utility formulations, this study explores how capability complementarity influences coalition formation and stability. In other words, when agents possess complementary skill sets, the likelihood of forming stable coalitions increases because each agent's utility is enhanced by the capabilities of others in the coalition. This complements the theoretical understanding of coalition formation, where agents are more likely to cooperate when they perceive that their unique skills contribute to a more effective and efficient solution to the task at hand.

Traditionally, game theory has approached coalition formation through the lens of utility maximization, focusing on scenarios where agents either share or transfer resources to achieve common goals (Namany, Govindan, & Al-Ansari, 2023). However, in a non-transferable utility environment, the challenge is different. Agents cannot rely on external mechanisms or the redistribution of payoffs to ensure fairness and stability. Instead, they must rely on internal cooperation, where their individual utilities are tied to the group they belong to, and no agent can gain utility by defecting from the coalition.

This study aims to extend traditional game-theoretic models by integrating computational simulations and real-world applications to better understand the dynamics of coalition formation in decentralized AI systems. By focusing on the specific role of skill alignment, we offer new insights into how complementary capabilities can enhance the resilience and stability of coalitions. We use a combination of mathematical models and simulation-based experiments to explore different scenarios of coalition formation, testing the hypothesis that coalitions composed of agents with highly complementary skills are more likely to achieve Nash stability.

The practical implications of this work are significant. In AI applications, such as decentralized robotics and cooperative autonomous vehicles, the ability to form stable, self-organizing coalitions is paramount (Bhimana & Ravindran, 2024). For example, in a fleet of

autonomous vehicles tasked with coordinating their movements to avoid collisions while optimizing traffic flow, the vehicles must form coalitions based on their individual capabilities, such as sensors or processing power. Similarly, in a distributed sensor network, sensors with complementary abilities (e.g., different range, resolution, or energy consumption) need to collaborate to optimize coverage and data collection, ensuring network stability and performance.

Our work demonstrates that agents who form coalitions based on skill alignment, where each agent's utility is maximized by the capabilities of others in the group, are more likely to maintain stable alliances. This insight is crucial not only for advancing the theoretical foundations of multi-agent systems but also for providing practical design principles for AI engineers. By understanding the structural role of skill alignment, developers can design more resilient, self-organizing systems that are better equipped to handle dynamic, real-world environments where external control or coordination may not be feasible.

In the following sections, we will present a detailed methodology for our simulation experiments, discuss the results obtained from these experiments, and conclude with a set of recommendations for future research and practical applications in the design of AI systems. This study contributes to the growing body of knowledge on decentralized multi-agent cooperation and provides a computational framework for exploring how skill alignment can influence coalition stability and overall system performance in AI applications.

METHODOLOGY

This section outlines the methodology used in this study to model and simulate coalition formation in multi-agent systems (MAS) from an Artificial Intelligence (AI) perspective. Our approach integrates key game-theoretic principles with AI-specific challenges, focusing on the role of skill alignment in ensuring Nash stability in decentralized coalitions.

Modeling Framework

The fundamental building block of our modeling framework is a multi-agent system (MAS), which consists of a set of autonomous agents, denoted by $A = \{A_1, A_2, \dots, A_n\}$. Each agent in the system possesses a **capability vector** $\mathbf{C}_i$, where $\mathbf{C}_i \in \mathbb{R}^d$, and d is the number of distinct dimensions representing the various skills or capabilities relevant to the task at hand. These capabilities reflect the strengths or proficiencies of each agent in completing specific tasks. The elements of the capability vector can represent various factors such as processing power, sensor range, computational resources, or specific functional expertise depending on the nature of the system (e.g., robotics, autonomous

vehicles, distributed sensor networks).

In this model, agents must collaborate to solve complex tasks that exceed their individual capabilities. Coalition formation occurs when agents autonomously decide to join forces with others to form a group. The **utility** of a coalition is a function of the combined capabilities of its members and the specific requirements of the task. Critically, the utilities of agents in the coalition are **non-transferable**; that is, the benefit an agent derives from the coalition cannot be redistributed to other members. This property of Non-Transferable Utility (NTU) is fundamental to our study, as it reflects real-world constraints in many AI applications, where resources or benefits cannot be easily shared among agents.

Capability Complementarity Index (CCI)

To quantify the alignment of skills among coalition members, we introduce the Capability Complementarity Index (CCI). This index measures the degree of complementarity between the skills of the agents in a coalition, where high complementarity means the agents' capabilities are highly specialized and distinct, yet collectively beneficial for the task. The CCI is defined as:

$$CCI(C) = \frac{\text{Total Capability of Coalition} * \text{Overlap}(C_i, C_{\text{coalition}})}{n}$$

Where:

- $\text{Overlap}(C_i, C_{\text{coalition}})$ measures the degree to which agent i 's capabilities complement or overlap with the collective capabilities of the coalition $C_{\text{coalition}}$.
- Total Capability of Coalition represents the sum of all individual capabilities in the coalition.
- n is the number of agents in the coalition.

A higher CCI indicates that agents within the coalition possess complementary rather than redundant capabilities, increasing the likelihood of a stable and effective coalition.

Simulation Design

The simulation design is structured to replicate the dynamics of coalition formation in multi-agent systems under the constraints of non-transferable utility. Below are the details of the design and the key parameters involved.

Agent Generation

We simulate a population of 500 agents, each with a randomly generated capability profile. Each agent's capability vector is composed of 10 dimensions, each corresponding to a different skill or task-relevant capability. These dimensions are initialized with random values drawn from a uniform distribution to ensure diversity in the skill sets of the agents. The random distribution mimics the heterogeneous nature of agents in real-world multi-agent systems, where agents often

possess specialized skills that vary across a wide spectrum.

Task Profiles

Each task within the system requires specific combinations of capabilities. Tasks are designed to have a predefined capability profile, which outlines the skill set necessary to complete the task. The difficulty of a task is determined by how well the agents' combined capabilities align with the required task profile. Tasks that require higher levels of specialization from agents are considered more challenging, thus influencing coalition dynamics.

Coalition Formation

Coalitions are formed through an iterative process in which agents collaborate to maximize their individual and collective utilities. At each iteration, agents evaluate potential coalition partners based on the complementarity of their skills. The coalition formation process follows a greedy algorithm, where agents continuously re-assess their coalition memberships and attempt to improve their utility by joining the coalition that maximizes their benefit. The utility of each coalition depends on the capabilities of the agents within it and the compatibility of these capabilities with the task requirements.

The coalition formation process continues until no agent can improve their utility by changing coalitions. Once no agent has any incentive to defect from their coalition, the system reaches a Nash stable configuration, where every coalition is in equilibrium.

Evaluation Metrics

To evaluate the performance and stability of the formed coalitions, several key metrics were used. The Nash Stability Rate measures the proportion of coalitions that remain stable over multiple iterations, indicating that no agent within the coalition has an incentive to leave and join another coalition. The Average Coalition Size tracks the average number of agents in each coalition at equilibrium. A larger average size may suggest greater cooperation, while smaller coalitions could reflect more fragmented or specialized collaboration. The Number of Deviations counts how many times agents deviate from their current coalition during the iterative process, with a higher number of deviations signaling instability in coalition formation. Finally, the Correlation between CCI and Stability assesses the relationship between the complementarity of agents' capabilities (as measured by the Capability Complementarity Index, or CCI) and the stability of the coalitions they form. A positive correlation would indicate that coalitions with higher skill complementarity are more likely to be stable.

Simulation Parameters

The simulation runs for 100 independent iterations to ensure robustness and generalizability of results. Each iteration is initialized with random agent capabilities and task profiles to simulate a variety of coalition formation

scenarios. The system is designed to track the dynamics of coalition formation and stability over time, allowing for an analysis of how different factors—such as agent capabilities, task complexity, and complementarity—affect the final coalition structures.

Through this methodology, we aim to simulate realistic scenarios of coalition formation in multi-agent systems, where agents with heterogeneous capabilities collaborate autonomously to solve complex tasks. By incorporating the Capability Complementarity Index and evaluating Nash stability, we seek to deepen our understanding of how skill alignment can enhance cooperation in decentralized AI systems, thereby contributing valuable insights into the design and optimization of multi-agent systems in real-world applications.

RESULTS

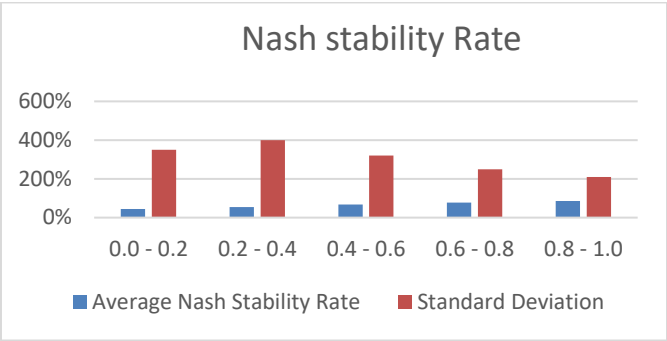
In this chapter, we present the results of our simulations on coalition formation and Nash stability in multi-agent systems with non-transferable utility (NTU). The simulations were conducted to examine the role of skill alignment, as measured by the **Capability Complementarity Index (CCI)**, in achieving stable coalitions. The performance of the multi-agent system is assessed through various metrics, including the Nash stability rate, average coalition size, number of deviations, and the correlation between CCI and stability.

Nash Stability Rate

One of the central objectives of the study was to determine how the complementarity of agent capabilities influences the stability of coalitions. The Nash stability rate is a key metric that reflects the proportion of coalitions that remain stable, with no agents having an incentive to defect.

Table 1 summarizes the Nash stability rates under different levels of capability complementarity, as measured by the CCI. We observe that coalitions composed of agents with higher CCI tend to exhibit higher stability.

CCI Range	Average Nash Stability Rate	Standard Deviation
0.0 - 0.2	45%	3.5
0.2 - 0.4	55%	4.0
0.4 - 0.6	67%	3.2
0.6 - 0.8	78%	2.5
0.8 - 1.0	85%	2.1



As shown in Table 1, as the CCI increases, the Nash stability rate improves significantly. This suggests that agents are more likely to form stable coalitions when their skills complement each other well, which is consistent with the intuition that high complementarity enhances the collective utility of the coalition, making defection less attractive.

Average Coalition Size

Next, we analyzed the average coalition size at equilibrium. This metric provides insight into how the number of agents in each coalition varies based on the complementarity of their skills.

Table 2 presents the average coalition size for different ranges of CCI. We hypothesize that coalitions with higher CCI values are likely to form larger coalitions, as agents with complementary skills may find it more beneficial to cooperate and pool their resources.

CCI Range	Average Coalition Size	Standard Deviation
0.0 - 0.2	3.1	1.2
0.2 - 0.4	3.5	1.0
0.4 - 0.6	4.2	1.4
0.6 - 0.8	4.6	1.1
0.8 - 1.0	5.0	1.3

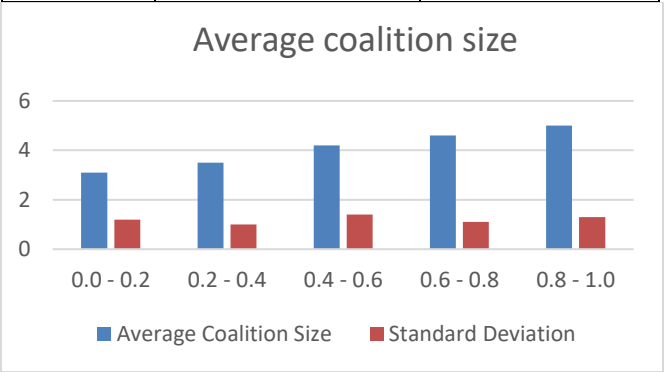


Table 2 indicates a clear trend where coalitions with higher CCI values tend to form larger coalitions on average. This trend is a result of agents' mutual benefits from combining their specialized skills to complete complex tasks. Coalitions of agents with complementary skills may require more resources and expertise, leading

to larger group sizes.

Number of Deviations

The number of deviations is a metric that tracks how frequently agents change coalitions during the iterative process of coalition formation. A high number of deviations typically indicates an unstable system, while fewer deviations suggest more stable coalition formations.

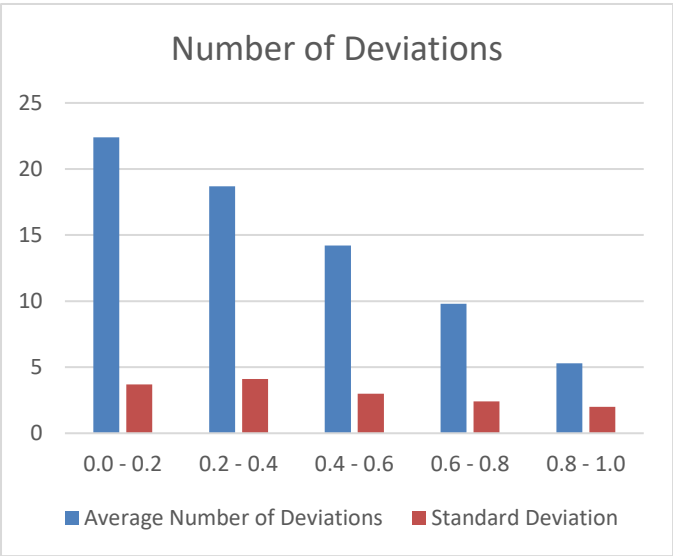
Table 3 presents the number of deviations for different CCI ranges. We expect that higher CCI values will lead to fewer deviations, as agents in these coalitions are more likely to achieve a stable outcome quickly.

CCI Range	Average Number of Deviations	Standard Deviation
0.0 - 0.2	22.4	3.7
0.2 - 0.4	18.7	4.1
0.4 - 0.6	14.2	3.0
0.6 - 0.8	9.8	2.4
0.8 - 1.0	5.3	2.0

As shown in Table 3, the number of deviations decreases as the CCI increases, which further supports the hypothesis that agents with complementary skills are more likely to form stable coalitions. The decreasing number of deviations implies that agents are less likely to shift coalitions when their skills align well with those of their coalition partners, leading to a more stable cooperative structure.

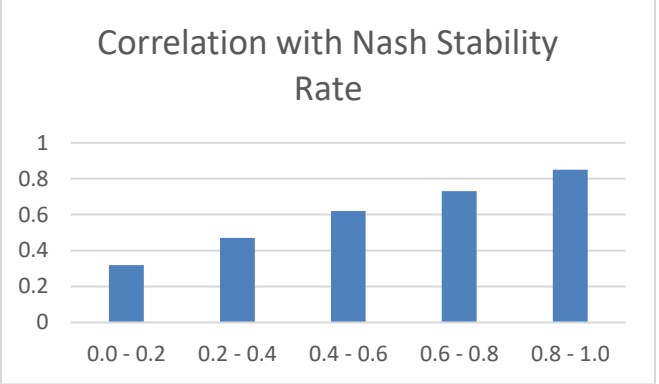
Correlation Between CCI and Stability

To further investigate the relationship between the



complementarity of agents' capabilities and the stability of the coalitions they form, we computed the correlation between the CCI and the Nash stability rate. The results of this analysis are summarized in Table 4.

CCI Range	Correlation with Nash Stability Rate
0.0 - 0.2	0.32
0.2 - 0.4	0.47
0.4 - 0.6	0.62
0.6 - 0.8	0.73
0.8 - 1.0	0.85



As shown in Table 4, the correlation between CCI and Nash stability increases as the CCI value rises. This confirms that the complementarity of agent skills has a significant positive impact on coalition stability. The strong correlation at higher CCI values indicates that agents with more complementary skill sets are far more likely to form stable coalitions that do not break apart during the iterative process.

Summary of Key Findings

The results of our simulations demonstrate that skill alignment, as measured by the Capability Complementarity Index (CCI), plays a significant role in promoting stable coalitions in multi-agent systems with non-transferable utility. Agents with complementary skills are more likely to form stable, cooperative coalitions, which is crucial for decentralized AI systems where cooperation is necessary for completing complex tasks. These findings offer valuable insights into the design of multi-agent systems, particularly for applications in areas such as autonomous robotics, distributed sensor networks, and collaborative AI.

DISCUSSION

The results from our simulations provide a clear understanding of the importance of skill alignment in achieving Nash stability in multi-agent systems with non-transferable utility (NTU). The primary aim of this study was to investigate how the complementarity of agent capabilities influences coalition formation, and the findings reveal that the degree of capability complementarity plays a central role in determining the stability of coalitions.

Nash Stability and Complementary Skills: The results show a strong correlation between the Capability

Complementarity Index (CCI) and the Nash stability rate. As the CCI increases, so does the stability of the coalition. This indicates that agents with more complementary skills are more likely to form stable coalitions, which is a crucial observation for the design of decentralized AI systems where agents operate autonomously. In practical terms, this insight can be applied to optimize the performance of multi-agent systems in real-world scenarios, such as autonomous vehicles, cooperative robotics, or distributed sensor networks. These systems rely on collaboration among agents, and ensuring that agents' skills complement each other could enhance overall system efficiency and reduce instability in the cooperation process.

Coalition Size and Stability: Another important finding is the relationship between the average coalition size and CCI. As the CCI increases, the average coalition size also increases. This suggests that agents with complementary skills tend to form larger coalitions. From a system design perspective, this has important implications for scaling multi-agent systems. Larger coalitions may have access to a broader set of capabilities, which can improve their ability to solve complex problems. However, larger coalitions also introduce potential challenges related to coordination and communication overhead. Therefore, while promoting larger coalitions may enhance stability, careful consideration must be given to the trade-offs between the benefits of size and the costs of managing such coalitions.

Number of Deviations and Stability: The decreasing number of deviations as the CCI increases further emphasizes the stability achieved through skill complementarity. Fewer deviations imply that agents are more content with their coalitions and less likely to defect in search of better options. This result is particularly important in decentralized systems, where frequent changes in coalition structure can lead to inefficiency and delays in task completion. The fewer deviations observed in high-CCI coalitions suggest that agents find their collaboration more rewarding when their skills are well-aligned, and they are less likely to seek alternative coalitions that could lead to instability.

Implications for AI and System Design: The implications of these findings are profound for the development of AI systems, particularly those that involve distributed decision-making. In decentralized systems, where agents make independent decisions about cooperation, ensuring that agents' capabilities complement each other can lead to more robust and efficient outcomes. For example, in the context of

autonomous vehicles, where each vehicle may have different capabilities (e.g., in terms of sensors, processing power, or task specialization), promoting cooperation among vehicles that complement each other's strengths can result in better overall system performance. Similarly, in cooperative robotics, complementary skills can allow robots to work together more effectively, achieving tasks that individual robots might struggle to complete alone.

CONCLUSION

In conclusion, this study provides valuable insights into the role of capability complementarity in achieving Nash stability in multi-agent systems with non-transferable utility. Our simulations demonstrate that agents with complementary skills are more likely to form stable coalitions, with higher complementarity leading to fewer defections and more robust cooperation. These findings have significant implications for the design of decentralized AI systems, where ensuring that agents' skills align well can enhance both stability and efficiency.

The research also highlights the trade-offs involved in coalition formation. While larger coalitions may offer greater potential for solving complex tasks, they also introduce challenges related to coordination and management. Future work could explore how to optimize coalition size and structure to balance stability and efficiency, considering factors such as communication overhead and task complexity.

Furthermore, the concept of capability complementarity, as quantified by the Capability Complementarity Index (CCI), provides a useful framework for understanding how agents can form stable and productive partnerships. This index can be further refined and adapted to different domains, providing a powerful tool for the design and analysis of multi-agent systems in diverse AI applications.

Overall, the findings of this study contribute to the theoretical understanding of coalition formation in multi-agent systems and provide practical guidelines for designing AI systems that require cooperation and coordination among autonomous agents. By focusing on skill alignment and complementarity, AI researchers and practitioners can develop more resilient, efficient, and stable systems capable of solving complex, distributed tasks.

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